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Bridge Network for Measuring Very Small Impedances from 4.2 to 300°K with a Null-Detector Sensitivity of 10^{-11} Volt

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An ac measuring technique devised primarily for measuring galvanomagnetic effects in metals is presented. The instrument may, however, be useful whenever it is desired to measure and record continuously impedances in the range 10^{-3} to 10^{-8} Ω. The sample assembly is disscussed in some detail. Measurements with the bridge show that in the temperature range 300°K to 4.2°K the noise level changes from 30×10^{-11} V to 10^{-11} V without any zero shift, and as a result the lower limit for the impedance range changes from 10^{-6} Ω to less than 5×10^{-8} Ω.

INTRODUCTION

In this paper we present an accurate and convenient method for measuring very small impedances.

The method has been developed with special regard to measurements of galvanomagnetic effects, i.e., magnetoresistance, transverse even effect, and Hall-effect in high purity single crystals of metals. Such measurements can give not only qualitative but also quantitative information regarding the Fermi surface of metals. The experimental method should give not only a relative accuracy (linearity and reproducibility) better than 0.1% but also an absolute accuracy better than 1%.

As a good and easily obtainable test of the quality of the crystal, it is desirable to measure the residual resistance ratio (RRR). To determine this resistance ratio precisely the resistance of the sample must be measured at room- and liquid-helium temperatures without changing the crystal mount, i.e., the bridge should work in the temperature range from room temperature down to 4.2°K. The resistance range required for covering this temperature range for most metals extends from, say 10^{-3} Ω to 10^{-8} Ω.

For practical reasons the sample current must not exceed 1 A. This produces voltages across the sample as small as 10^{-8} V. Thus, the null-detector must have a sensitivity of at least 10^{-11} V.

METHOD

In principle a sensitivity of 10^{-11} V can very easily be achieved, since thermal noise in 10^{-8} Ω at 4.2°K amounts to no more than 10^{-15} V/cps$^{\frac{1}{2}}$. The problem then is the matching of the crystal impedance to the input impedance of an ordinary amplifier.

This impedance matching is improved by means of transformers. This implies that measurements must be carried out by alternating current of a suitable frequency. Since skin-effect should not impair the measurements, the frequency must be kept suitably low. On the other hand, the frequency should be kept sufficiently high to reduce $1/f$ noise in the amplifier. We have used frequencies up to 32 cps without measureable increase in resistance due to skin effect.

To fulfil linearity and accuracy requirements, a bridge circuit must be employed. The impedance of the sample circuit must be kept very low in order not to impair the sensitivity of the null detector.

Construction of a resistor with known resistance operating from 4.2°K to 300°K seems to be a difficult task; however, since an ac method is used, a mutual inductance may be employed as the reference impedance. By proper construction of a coil former a mutual inductance can be made almost independent of temperature.

Another advantage of ac methods over dc methods is that the null detector does not respond to thermal emf's. A disadvantage is the inductive coupling from the sample to detector circuit.

SAMPLE ASSEMBLY

The circuit inside the dotted line (Fig. 1) shows the connections of the sample assembly. The sample resistance between the voltage terminals is denoted R_x (in fact R_x may also represent a transverse voltage coefficient). The above-mentioned inductive coupling may be represented by a self-induction L_x in series with R_x. The voltage delivered from the automatic balancing apparatus is denoted αE_0 (α real). Assuming sample assembly impedances small compared to R_N and R_L parallel to R_k and assuming an infinite gain in the operational amplifier, the equilibrium conditions may be written

$$R_x = -\alpha R_N(M/R_k rC),\ L_x = M(R_N/R_L).$$

The error voltage is fed to an input amplifier via two cascaded transformers with a total turns ratio of 10^4.

The mechanical layout of the sample assembly is shown in Figs. 2 and 3.

First error voltage transformer consists of the "coils" L_1 and L_2. The mutual inductance between coils L_3–L_4 is denoted M. L_1 and L_3 are formed by two concentric copper tubes with a conductor cross section of 15 mm². The tubes are connected to each other at one end and connected to the sample at the other end. The electrical connections are made by soldering very short pieces of 0.4 mm diam copper wire from the sample to the copper tubes.

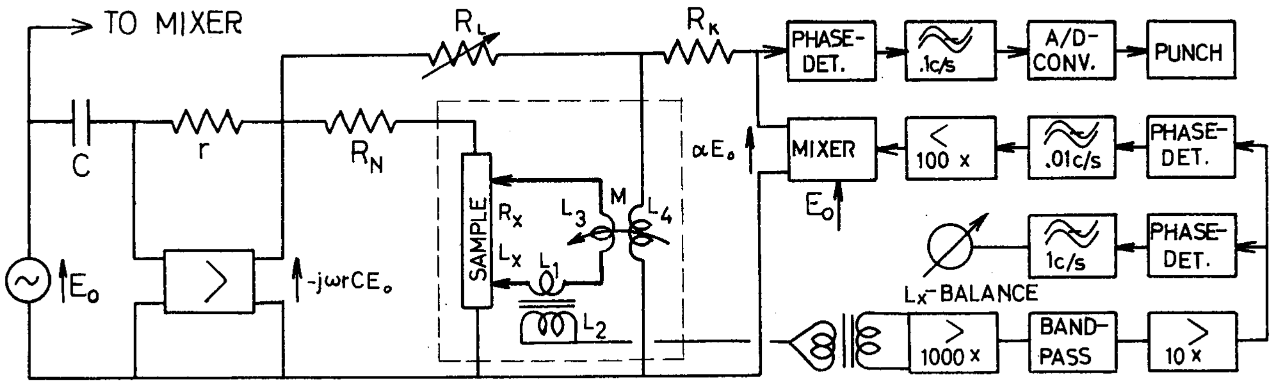

FIG. 1. Bridge network with associated balancing equipment.

The coil L_4 consists of 265 turns of 0.06 mm diam copper wire wound toroidally on a precision machined nonmagnetic coil former placed in the space between the copper tubes and concentric with the tubes (dimensions: 6.40 mm i.d., 8.80 mm o.d., length 36.2 mm).

Taking the wire thickness into account, the mutual inductance M between the coils L_3 and L_4 can be calculated as 0.645 μH at 300°K. At 4.2°K the mutual inductance is about 0.5% lower owing to thermal contraction of the coil former.

The secondary L_2 of the error voltage transformer consists of approximately 100 turns of 0.09 mm diam copper wire around a toroidal mumetal core (core dimensions: 7.0 mm i.d., 8.0 mm o.d.. length 3.2 mm; relative permeability at 4.2°K: 5000—10 000).

Care must be taken that the dc magnetic field is sufficiently low not to saturate the mumetal core. (The core is shielded by a soft iron ring and itself guarded from saturation by placing it where the magnetic field from the superconducting magnet is zero.)

Due to the cylindrical geometry of the sample assembly,

(1) the resistance in the sample circuit can be kept as low as $2\times10^{-6}\ \Omega$,

(2) the mutual inductance can be calculated exactly, and

(3) the bridge circuit is very insensitive to stray magnetic fields (rotation symmetry).

In order to measure the magnetoresistance and the transverse voltage simultaneously, the sample assembly is made up of two identical circuits, with one common potential probe.

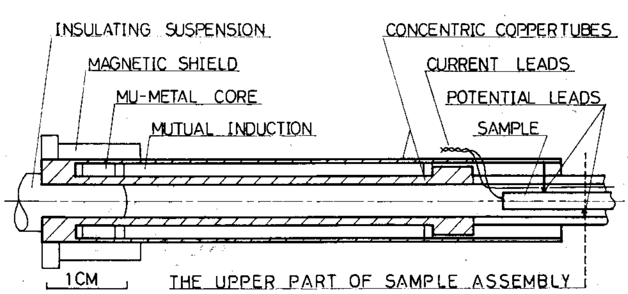

FIG. 2. Sample assembly.

AUTOMATIC BALANCING EQUIPMENT

The automatic balancing equipment is shown as the block diagram part of Fig. 1. The error voltage is amplified, passed through two orthogonally operating phase-sensitive detectors and filtered, producing two direct voltages. If the phase-sensitive detectors operate at a suitable phase relative to E_0, one voltage will represent "L_x balance," the other "R_x balance."

The voltage representing the L_x balance is set to zero by means of R_L; this is done manually since, as can be seen from the equilibrium conditions, R_L does not change with R_x.

The voltage representing R_x balance is amplified and fed to one input of a mixer. E_0 is fed to the other input of the mixer.

The output from the mixer is a voltage almost proportional to the product of the two input voltages. This voltage is denoted αE_0 where α is a real factor depending on (in fact almost proportional to) the direct voltage on the first input. αE_0 is then introduced into the sample circuit through the mutual inductance M.

The low-pass filter in the R_x branch has an upper frequency of 0.01 cps to ensure stability.

Open loop gain is normally kept at 200 corresponding to 46 dB.

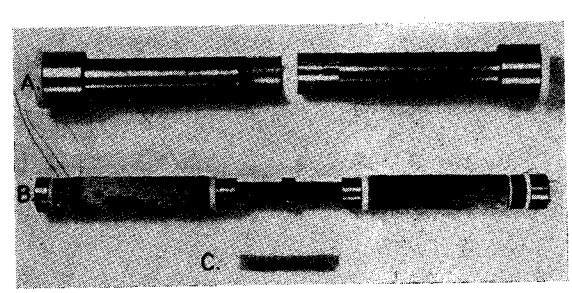

FIG. 3. Parts for sample assembly. A—outer copper tubes with magnetic shields. B—inner copper tube with mutual inductances and mumetal transformers. C—Sample.

E_0 is kept constant during the measurements; αE_0 may then be taken as a measure of α. The voltage αE_0 is rectified, filtered, and passed through an analogue/digital converter to a paper tape puncher.

AUXILIARY APPARATUS

Among the auxiliary apparatus we mention a superconducting magnet built as a split coil solenoid yielding a horizontal field of maximum strength 35 kG. The sample mounted in the assembly is rotated about a vertical axis in this field. Using a step motor for rotation it is possible to collect the data on paper tape through an A/D converter or else on strip chart recorders. Although the data taken represent discrete points, the results are presented as continuous curves because the angular steps are chosen sufficiently small to ensure that no detail of the curve is missed.

Data from the two channels and for both directions of the magnetic field are collected for further arithmetic treatment. This treatment, which resolves the data in effects even and odd in the magnetic field, cannot be carried out with pure analogue circuits. For that reason it is most convenient to use a digital data collecting system. The data tape is processed by a digital computer which reduces the raw data, plots the final curves on a digital plotter, and lists the extremal direction cosine for open orbit directions, etc., in tables. Such a system not only reduces the manual work enormously but in addition it allows a resolution not possible without an automatic data collecting system.

RESULTS

Figure 4 shows a result obtained by the bridge described above. The sample was spark cut from a large crystal of 99.999% copper giving a sample with RRR=800. The rotation diagram was taken at 31 kG in a transverse rotation about an axis near the ⟨511⟩ direction. Curve 4(A) shows the magnetoresistance, 4(B) the transverse even effect, and 4(C) the transverse odd effect (Hall effect) as functions of magnetic field orientation. The peaks of the magnetoresistance and of the transverse even effect curves are proportional to the square of the magnetic field and occur at orientations of the magnetic field for which a sizeable number of unidirectional open orbits exist.

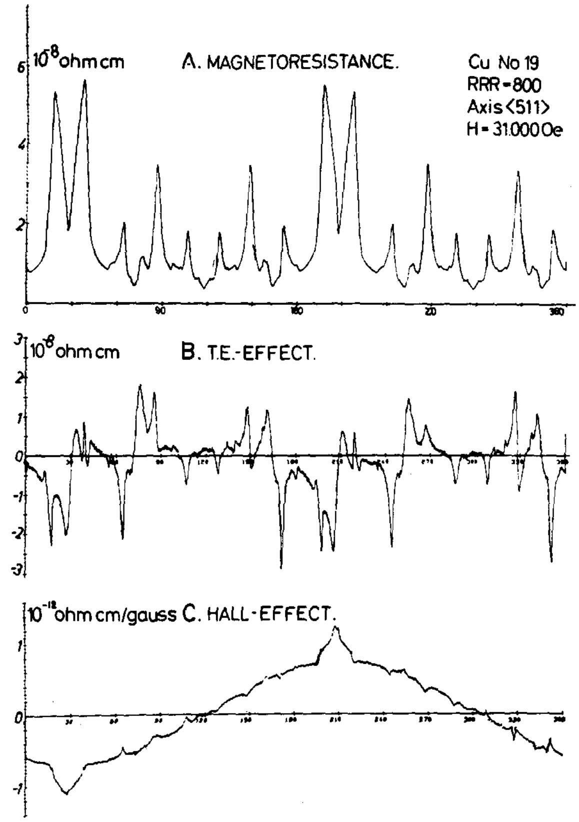

FIG. 4. Galvanomagnetic effects in copper obtained with the bridge. Resistivity (A), transverse even effect (B), and Hall effect (C) versus magnetic field orientation. The current direction and rotation axis are the ⟨511⟩ direction and the transverse probe direction is ⟨$\bar{2}$55⟩.

During these measurements the bridge has been applied in the temperature range 4.2°K to 300°K without zero shift, needing only open loop gain corrections. The impedance range used until now has been from 10^{-3} Ω and down to 5×10^{-8} Ω at 4.2°K. The noise level at 4.2°K was 10^{-11} V referred to a bandwidth of 0.1 cps. At 300°K the noise level was about 30 times higher and the lower impedance limit raised accordingly.

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